Intersection of a Line to a Plane

A Plane is defined as:

$$(i) \qquad Ax + By + Cz + D = 0$$

Where, given 3 points on that Plane (x1,y1,z1), (x2,y2,z2) & (x3,y3,z3):

A = y1(z2 - z3) + y2(z3 - z1) + y3(z1 - z2)
B = z1(x2 - x3) + z2(x3 - x1) + z3(x1 – x2)
C = x1(y2 – y3) + x2(y3-y1) + x3(y1 – y2)
-D = x1(y2z3 – y3z2) + x2(y3z1 – y1z3) + x3(y1z2 – y2z1)

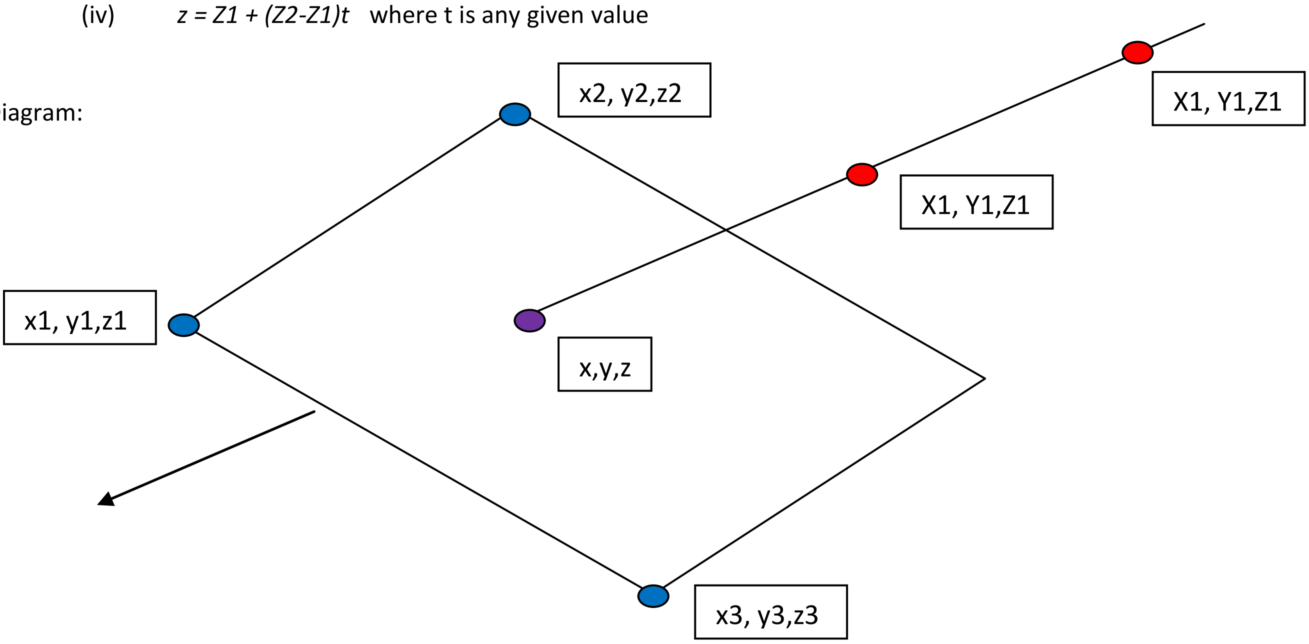
Then, D = -1(-D)

A Line may be defined by 2 points along that Line (X1, Y1, Z1) & (X2, Y2, Z2)

The co-ordinates of any point on that Line may be given as:

(ii) $x = X1 + (X2-X1)t$
(iii) $y = Y1 + (Y2-Y1)t$
(iv) $z = Z1 + (Z2-Z1)t$ where t is any given value

Diagram:



Substituting (ii), (iii) & (iv) into (i) gives:

(v) $A(X1+(X2-X1)t) + B(Y1+(Y2-Y1)t) + C(Z1+(Z2-Z1)t) + D = 0$

Expanding and solving for "t":

A(X1)+A(X2-X1)t+ B(Y1)+B(Y2-Y1)t+C(Z1)+C(Z2-Z1)t+ D = 0
-t = (A(X1)+B(Y1)+C(Z1)+D) / (A(X2-X1)+B(Y2-Y1)+C(Z2-Z1))
t = -1 (-t)

The Intersection co-ords (x,y,z) are calculated by replacing the value of "t" into equations (ii), (iii) & (iv)

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